

Strand7 Software

# Theoretical Manual

*Theoretical background to the Strand7 finite element analysis system*



 Strand7<sup>®</sup>

[www.strand7.com](http://www.strand7.com)

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# Preface

The finite element method (FEM) is today the most powerful numerical tool available for the analysis of structures. The method is used extensively in virtually all the engineering disciplines, not only for the analysis of structures and other engineering systems, but increasingly to drive the design.

Strand7 is a software system based on the finite element method. It provides a visual environment for applying the method to real world engineering problems.

This manual is not intended to be a comprehensive review of the finite element method. There are thousands of books, journals and other resources dedicated to the concepts, the theory and the applications of the method (e.g. see the bibliography). Instead, this manual aims to provide Strand7 users with relevant theoretical information about the finite element method as it applies to Strand7. This should give users further insight into the behaviour of their Strand7 models and therefore better interpretation of analysis results; it should also encourage sensible use of Strand7 by highlighting the embedded assumptions and limitations of both the method and its implementation. The manual attempts to present its material in a simple and straightforward manner, focusing on the physical meaning of the subject matter rather than the highly mathematical one.

The reader is assumed to have some knowledge of statics, dynamics, mechanics of materials, the theory of elasticity and stress analysis. Familiarity with numerical analysis, matrix computation and the basics of the finite element method would be advantageous. A basic understanding of matrix notation is also assumed.

This manual should be used in conjunction with other Strand7 documentation such as the *Using Strand7* handbook, the *Strand7 Verification Manual*, and the *Strand7 Online Help*.



# Chapter Outline

Chapter 1 gives a brief summary of the finite element formulation covering the basic concepts of the theory. It provides basic information for a relatively new user of the finite element method and introduces concepts referred to in the following chapters.

Chapter 2 describes the linear material models available in Strand7. The models range from generalized Hooke's material models for isotropic, orthotropic and anisotropic materials to laminate material models and user-defined material models for plate/shell elements. This chapter concludes with a description of the way Strand7 treats temperature-dependent and time-dependent material properties.

Chapter 3 gives a detailed description of all the available line elements, referred to as beams in Strand7. Strand7 provides a variety of different line elements for various applications, such as spring-damper elements for static and dynamic analysis, catenary elements for cable structures, contact elements for contact problems or changing boundary conditions, pipe elements for pipe systems, thin and thick beam elements, and user-defined beam elements.

Chapter 4 gives a detailed description of all the surface elements, referred to as plates in Strand7, including 3-node and 6-node triangular elements, and 4-node, 8-node and 9-node quadrilateral elements. These elements can be used for plane stress and plane strain analysis, axisymmetric solid analysis, plate and shell analysis, 3D membrane analysis and heat transfer analysis.

Chapter 5 gives a detailed description of all the three-dimensional (3D) solid elements in Strand7, including 4-node and 10-node tetrahedral elements, 5-node and 13-node pyramid elements, 6-node and 15-node wedge elements, and 8-node, 16-node and 20-node hexahedral elements. These elements are used for 3D continuum problems in structural and heat transfer analysis.

Chapter 6 discusses links, including the physical function of the different links, their corresponding mathematical expressions and implementation within Strand7. The available link types include master-slave, sector symmetry, coupling, pinned, rigid, shrink and 2-point links.

Chapter 7 introduces the Strand7 solvers. These solvers can be used to analyse a variety of problems encountered in industry and research. For instance, they can be used for static analysis (linear and nonlinear), buckling analysis, dynamic analysis (eigenvalue problems, harmonic response, spectral response, linear and nonlinear transient) and heat transfer analysis (steady-state and transient).

Chapter 8 introduces topics on structural dynamics, including types of dynamic loading, modelling of structural mass and damping, time domain integration, the mode superposition method, and base excitation. These topics relate to a number of the Strand7 solvers including natural frequency, harmonic response, spectral response and transient analysis.

Chapter 9 presents the basic theory of heat transfer, including conduction, convection and radiation (the three modes of heat transfer supported by Strand7), the finite element formulation of heat transfer and the application of the heat solver to other types of field problems.

Chapter 10 introduces material nonlinearity. Material models for elasto-plastic, nonlinear elastic and rubber materials are described, together with the Duncan-Chang model for nonlinear analysis of soils.

Chapter 11 introduces geometric nonlinearity. It covers the basic concepts of large displacement and finite (large) strain, the formulation of the geometric stiffness matrix, solution strategies, strain measures and the use of the geometric nonlinear solver for buckling and post-buckling analysis.

Chapter 12 describes some of the numerical procedures that are employed in Strand7, most of which are essential for any software implementation of the finite element method. This includes matrix decomposition, eigenvalue extraction, time-domain integration for transient dynamic problems, modified Newton-Raphson iteration for solving nonlinear equations, matrix static condensation, numerical integration schemes for the element matrix calculations and the Fast Fourier Transform.

Chapter 13 covers the procedures used in Strand7 for the post-processing of the results, including result extrapolation and averaging, contouring, display of deformed elements and sub-modelling.

Chapter 14 discusses various special topics including projected load, structural and non-structural mass, tables and their application, bandwidth minimization of the global stiffness matrix and node reordering methods, the treatment of drilling degrees of freedom and local singularity suppression, and the calculation of the torsion constant and shear area for beam cross-sections.

Appendix A summarizes the node and element attributes supported by Strand7 and the way they are used by the solvers.

Appendix B is a guideline to finite element modelling including a very useful checklist for the finite element modelling process in practice.

Appendix C summarizes stress and strain conventions, transformations, principal stresses, stress invariants, deviatoric stresses and effective stresses.

Appendix D is a bibliography that lists some of the important books and articles on the topic of finite element analysis.